

QUANTUM BIRTH OF A HOT UNIVERSE

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Abstract

We consider quantum birth of a hot Universe in the framework of quantum geometrodynamics in the minisuperspace model. The energy spectrum of the Universe in the pre-de-Sitter domain naturally explains the cosmic microwave background (CMB) anisotropy. The false vacuum where the Universe tunnels from the pre-de-Sitter domain is assumed to be of a Grand Unification Theory (GUT) scale. The probability of the birth of a hot Universe from a quantum level proves to be about $10^{-10^{14}}$. In the presence of matter with a negative pressure (quintessence) it is possible for open and flat universes to be born as well as closed ones.

1 Introduction

In the framework of the standard scenario [1, 2] a quantum birth of the Universe [3, 4], as a result of tunnelling [5], is followed by a classical decay of the de Sitter (false) vacuum with the equation of state $p = -\varepsilon$ into a hot expanding Universe called the Big Bang. One of the proofs of the hot Universe model is a discovery of the CMB with the temperature about $3K$ [6]. G. Gamow, the author of the tunnel effect [7] basic to the Universe tunnelling, was the first to predict CMB [8] being a radiation of the hot

Universe cooled due to its expansion. Recently a CMB anisotropy $\frac{\Delta T}{T} \simeq 10^{-5}$ has been discovered [9]. This anisotropy allows a large-scale structure to be explained.

In the present paper we consider a quantum model of the hot Universe. In the pre-de-Sitter domain radiation energy levels are quantized, which allows temperature fluctuations to be treated as a manifestation of a quantum behaviour of the Universe before its birth from the false vacuum. In the previous papers [10, 11] as well as in [12, 13] the presence of a nonzero energy in Schrödinger's equation was shown to be due to radiation or ultrarelativistic gas. Here we shall calculate the temperature of this radiation as well as the probability of its tunnelling through the barrier separating the pre-de-Sitter domain from the false vacuum. The quantized temperature is compared with the observed CMB anisotropy. We assume that the false vacuum energy density is at the GUT scale.

2 Approach

The Wheeler-DeWitt equation for Friedmann's world reads [10]

$$\frac{d^2\psi}{da^2} - V(a)\psi = 0 \quad (1)$$

where

$$V(a) = \frac{1}{l_{pl}^4} \left(ka^2 - \frac{8\pi G \varepsilon a^4}{3c^4} \right), \quad (2)$$

a is the scale factor, $k = 0, \pm 1$ is the model parameter.

As follows from Einstein's equations, the energy density may be written in the form

$$\varepsilon = \varepsilon_0 \sum_{n=0}^6 B_n \left(\frac{r_0}{a} \right)^n. \quad (3)$$

Here B_n are contributions of different kinds of matter to the total energy density at the de Sitter horizon scale.

$$n = 3(1 + \alpha) \quad (4)$$

where α is the parameter characterizing the equation of state

$$p = \alpha \varepsilon. \quad (5)$$

For the equations of state satisfying the weak energy dominance condition

$$|p| \leq \varepsilon, \quad \varepsilon > 0 \quad (6)$$

we have:

- $n = 0$ ($\alpha = -1$) for the de Sitter (false) vacuum,
- $n = 1$ ($\alpha = -\frac{2}{3}$) for domain walls,
- $n = 2$ ($\alpha = -\frac{1}{3}$) for strings,
- $n = 3$ ($\alpha = 0$) for dust,
- $n = 4$ ($\alpha = \frac{1}{3}$) for radiation or ultrarelativistic gas,
- $n = 5$ ($\alpha = \frac{2}{3}$) for perfect gas,
- $n = 6$ ($\alpha = 1$) for ultrastiff matter.

Matter with a negative pressure (in particular, corresponding to the equations of state for false vacuum, domain walls and strings) has been recently called quintessence [14] due to which the Universe expands with acceleration.

The de Sitter horizon is defined as

$$\frac{1}{r_0^2} = \frac{8\pi G\varepsilon_0}{3c^4}. \quad (7)$$

Since $\varepsilon = \varepsilon_0$ at $a = r_0$, we obtain

$$\sum_{n=0}^6 B_n = 1. \quad (8)$$

Separating in the potential (2) a term independent of the scale factor, we reduce the Wheeler-DeWitt equation to Schrödinger's

$$-\frac{\hbar^2}{2m_{pl}} \frac{d^2\psi}{da^2} - [U(a) - E]\psi = 0 \quad (9)$$

for a planckeon with the energy E , corresponding to radiation, moving in the potential created by other kinds of matter. The potentials in Wheeler-DeWitt's and Schrödinger's equations are related by the formula

$$V(a) = \frac{2m_{pl}}{\hbar^2} [U(a) - E]. \quad (10)$$

Restricting ourselves to radiation, strings and the de Sitter vacuum, we obtain

$$U(a) = \frac{m_{pl}c^2}{2l_{pl}^2} [(k - B_2)a^2 - \frac{B_0a^4}{r_0^2}], \quad (11)$$

$$E = \frac{m_{pl}c^2}{2} \left(\frac{r_0}{l_{pl}} \right)^2 B_4. \quad (12)$$

The energy E is related to the contribution of radiation to the total energy density.

3 WKB Calculation of the Energy Spectrum and Penetration Factor

The quantization of energy in the well (a Lorentzian domain of the pre-de-Sitter Universe) follows the Bohr-Sommerfeld formula [15]

$$2 \int_0^{a_1} \sqrt{2m_{pl}(E - U)} da = \pi \hbar \left(n + \frac{1}{2} \right), \quad (13)$$

where $U(a_1) = E$, $n = 1, 3, 5, \dots$ (since $\psi(0) = 0$ if $U = \infty$ for $a < 0$), and the penetration factor for the Universe tunnelling through the potential barrier between the pre-de-Sitter and de Sitter domains is given by Gamow's formula

$$D = \exp \left(-\frac{2}{\hbar} \left| \int_{a_1}^{a_2} \sqrt{2m_{pl}(E - U)} da \right| \right) \quad (14)$$

where $U(a_1) = U(a_2) = E$.

Mathematically, the problem reduces to evaluation of $\int \sqrt{2m_{pl}(E - U)} da$ where the energy and the potential satisfy formulae (11), (12) respectively. The potential (11) has a minimum $U = 0$ at $a = 0$, a maximum $U = \frac{m_{pl}c^2}{8B_0}(k - B_2)^2 \left(\frac{r_0}{l_{pl}} \right)^2$ at $a = r_0 \sqrt{\frac{k - B_2}{2B_0}}$ and zeros at $a = 0$ and $r_0 = \sqrt{\frac{k - B_2}{B_0}}$, where $k - B_2 > 0$ and $B_0 > 0$.

Near the minimum we have

$$U = U(0) + \frac{1}{2} \frac{d^2 U}{da^2} \Big|_{a=0} \cdot a^2 \quad (15)$$

for $a \ll r_0 \sqrt{\frac{k - B_2}{B_0}}$.

Near the maximum we have

$$U = U(a_{max}) + \frac{d^2 U}{da^2} \Big|_{a=a_{max}} \cdot (a - a_{max})^2 \quad (16)$$

for $|r_0\sqrt{\frac{k-B_2}{2B_0}} - a| \ll r_0\sqrt{\frac{k-B_2}{2B_0}}$.

Formulae (13) and (14) take the same value

$$U_{med} = \frac{m_{pl}c^2}{2}(k - B_2)^2 \left(\frac{r_0}{l_{pl}}\right)^2 \left[\frac{1}{(1 + \sqrt{2})^2} - \frac{1}{(1 + \sqrt{2})^4} \right] \quad (17)$$

at

$$a_{med} = \frac{r_0}{1 + \sqrt{2}} \sqrt{\frac{k - B_2}{B_0}}. \quad (18)$$

Hence $U_{med} = 4\left[\frac{1}{(1+\sqrt{2})^2} - \frac{1}{(1+\sqrt{2})^4}\right]U_{max} \approx 0.569U_{max}$ at $a_{med} = \frac{\sqrt{2}}{1+\sqrt{2}}a_{max} \approx 0.586a_{max}$. Thus we may use formula (15) for $a \leq 0.586a_{max}$ and $U \leq 0.569U_{max}$ and formula (16) for $a \geq 0.586a_{max}$ and $U \geq 0.569U_{max}$.

Using formulae (13) and (15), we calculate the energy spectrum

$$E = m_{pl}c^2 \sqrt{k - B_2} \left(n + \frac{1}{2}\right) \quad (19)$$

where

$$n + \frac{1}{2} < \frac{(k - B_2)^{3/2}}{8B_0} \left(\frac{r_0}{l_{pl}}\right)^2 \quad (20)$$

since $E < U_{max}$. Although formula (19) has been obtained in the WKB approximation, it coincides with the exact solution for a harmonic oscillator considered previously for the case $r_0 = l_{pl}$ [10].

Using formulae (14) and (16), we calculate the penetration factor near the maximum of the potential

$$D = \exp \left\{ -\pi \left(\frac{r_0}{l_{pl}}\right)^2 \frac{|\frac{(k-B_2)^2}{4B_0} - B_4|}{\sqrt{2(k-B_2)}} \right\}. \quad (21)$$

Although the problem of penetration through a barrier near its maximum was considered by other authors [15, 16], our approach gives a more exact formula because we do not expand $\sqrt{2m_{pl}(E - U)}$ in series for a parabolic potential and then calculate $\int \sqrt{2m_{pl}(E - U)} da$ but calculate this integral directly.

For $B_4 \ll \frac{(k-B_2)^2}{4B_0}$ the penetration factor (21) reduces to

$$D = \exp \left\{ -\frac{2}{3} \frac{(k - B_2)^{3/2}}{B_0} \left(\frac{r_0}{l_{pl}}\right)^2 \right\}. \quad (22)$$

Formulae (21) and (22) satisfy the WKB approximation as $\left(\frac{r_0}{t_{pl}}\right)^2 \gg 1$. As seen from them, open and flat universes can be born if $k - B_2 > 0$, i.e for $B_2 < 0$, in other words for quintessence with a negative energy density.

4 Cosmic Microwave Background Temperature and Anisotropy. Probability of the Birth of a Hot Universe

In the hot Universe model the energy density of radiation and ultrarelativistic gas is given by the formula [17]

$$\varepsilon = \frac{3c^2}{32\pi G t^2}. \quad (23)$$

On the other hand for the matter with the equation of state $p = \frac{\varepsilon}{3}$ we have [2]

$$\varepsilon = \frac{4}{c} \sigma \Theta^4 N(\Theta) \quad (24)$$

where $\sigma = \frac{\pi^2}{60\hbar^3 c^2}$, Θ is the temperature in degrees T multiplied by the Boltzmann constant (the average energy of a particle $\bar{E} = 3\Theta$), $N(\Theta) = 10^2 - 10^4$ is assumed to be determined from observations. From formulae (23), (24) we obtain [18]

$$\Theta = \sqrt[4]{\frac{45}{32\pi^3 N(\Theta)}} m_{pl} c^2 \sqrt{\frac{t_{pl}}{t}} \quad (25)$$

where $N(\Theta) = 4.07 \cdot 10^3$, which gives $T = 2.73$ K for $t = 1.5 \cdot 10^{10}$ yr (the Hubble constant $H_0 = 65 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$).

Assume that the false vacuum energy density is related to Grand Unification scale E_{GUT} which can be described by the formula [19]

$$E_{GUT} = m_p c^2 e^{\frac{\hbar c}{4e^2}} = 7.03 \cdot 10^{14} \text{ GeV} \quad (26)$$

where m_p is the proton mass. Substituting (26) into (25), and taking account $3\Theta(t_0) = E_{GUT}$, we obtain $t_0 = 4.86 \cdot 10^{-37}$ s and $r_0 = 2.92 \cdot 10^{-26}$ cm.

The energy (19) is the energy density (24) multiplied by the volume $\frac{4\pi}{3}r_0^3$. It gives us the quantized temperature

$$\Theta = \sqrt[4]{\frac{45}{4\pi^3 N(\Theta)}} \sqrt[8]{k - B_2} \left(\frac{l_{pl}}{r_0}\right)^{3/4} m_{pl} c^2 \left(n + \frac{1}{2}\right)^{1/4}. \quad (27)$$

The average energy of a particle is estimated within the range

$$1.42 \cdot 10^{13} \text{GeV} \leq \bar{E} \leq 3.24 \cdot 10^{16} \text{GeV}$$

for

$$\frac{3}{2} \leq n + \frac{1}{2} \leq \frac{(k - B_2)^{3/2}}{8B_0} \left(\frac{r_0}{l_{pl}}\right)^2.$$

The lowest energy is close to the values predicted by reheating models, the highest one, being the most probable, is of the order of the monopole rest energy [2]. Thus the model predicts existence of monopoles at the beginning of inflation which dilutes their density to the required level.

On the other hand, equating (12) to (19), we obtain

$$B_4 = 2\sqrt{k - B_2} \left(\frac{l_{pl}}{r_0}\right)^2 \left(n + \frac{1}{2}\right). \quad (28)$$

CMB temperature fluctuations are given by the formula

$$\frac{\Delta T}{T} = \frac{\sqrt[4]{n + \frac{3}{2}} - \sqrt[4]{n + \frac{1}{2}}}{\sqrt[4]{n + \frac{1}{2}}}. \quad (29)$$

For $n \gg 1$ we have

$$\frac{\Delta T}{T} \approx \frac{1}{4n}. \quad (30)$$

For $n = 2.5 \cdot 10^4$ we have $\frac{\Delta T}{T} = 10^{-5}$. From formula (27) at $k = 1$, $B_2 = 0$ we obtain the average energy $\bar{E} = 3\Theta = 1.61 \cdot 10^{14}$ GeV being of the order of the Grand Unification energy which is not known exactly. Its estimates vary, say, from $1.9 \cdot 10^{14}$ GeV [20] to $7.5 \cdot 10^{15}$ GeV [21]. We have chosen the estimate (26) within this range which leads to the CMB temperature fluctuations comparable with the observed CMB anisotropy values.

For $n = 2.5 \cdot 10^4$, $r_0 = 2.92 \cdot 10^{-26}$ cm, $k = 1$, $B_2 = 0$ from formula (28) we have $B_4 = 1.53 \cdot 10^{-10} \ll 1$, hence $B_0 \approx 1$ due to formula (8). Since

$B_4 \ll \frac{k-B_2}{4B_0}$, we can use formula (22) to calculate the probability of the birth of a hot Universe, which gives $D = e^{-2.18 \cdot 10^{14}}$.

The model predicts a quantum birth of the GUT-scale hot Universe with the temperatures about those predicted by reheating models and, as a consequence, the observed CMB anisotropy and plausible amount of monopoles.

5 Conclusion

We have considered a possibility of quantum birth of a hot Universe avoiding the reheating stage. Open and flat universes can be also created due to quintessence with a negative energy density. The model based on the GUT-scale false vacuum naturally explains CMB anisotropy and predicts monopole existence in terms of the initial quantum spectrum of the Universe in the pre-de-Sitter domain. Thus quantum cosmology proves to have direct observational consequences, and GUT acquires evidence in its support.

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